COMMON PRE-BOARD EXAMINATION 2022-23

Class XII

Marking Scheme

Subject: APPLIED MATHEMATICS - 241

Q. No.	Hints/Solution	Marks
	SECTION – A	
	(Multiple Choice Questions)	
	(Each question carries 1 mark)	
1.	Option (b) 21	1
2.	Option (b) 4	1
3.	Option (d) 0.34	1
4.	Option (a) N – 1	1
5.	Option (a) 3.6 km	1
6.	Option (b) parameter	1
7.	Option (c) 7.2 minutes	1
8.	Option (a) ₹ 3000	1
9.	Option (c) $\frac{x^3}{3} - e^x + C$	1
10.	Option (a) 8000	1
11.	Option (b) 5 corner points including (7,7) and (3,3)	1
12.	Option (a) 56.5%	1
13.	Option (c) 3	1
14.	Option (b) 5000	1
15.	Option (b) 20:11	1
16.	Option (c) 1.48	1
17.	Option (c) 19,25,31	1
18.	Option (b) Parts of a year	1
19.	Option (b) Both A and R are true but R is not the correct explanation of the assertion	1
20.	Option (b) Both A and R are true but R is not the correct explanation of the assertion	1
	SECTION – B	
	(All questions are compulsory. In case of internal choice, attempt any one question only)	
21.	Here R=2500	½
	$i = 0.03$	
	$P=R+\frac{R}{i}$	½
	$P=2500 + \frac{2500}{0.03}$	½
	$=2500 + 83333.33 =$ ₹ 85833.33	½
22.	$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	
	$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$	1
	$k = 2$	1

OR

Adj(A) = $\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ 1

$A^{-1} = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ 1

23. Let number of necklaces and bracelets produced by firm per day be x and y, respectively.

Maximize: Z = 100x + 300y 1

subject to, $x + y \leq 24$ $x + 2y \leq 32$ and $x, y \geq 0$ 1

24. Length of course = 500 meters

Time taken by B to cover by 60 meters = 12 seconds.

$\therefore$ time taken by B to cover the course = $\frac{12}{60} \times 500 = 100$ seconds 1

$\therefore$time taken by A to cover the course= 100 − 12 seconds = 88 seconds = 1 minute 28 seconds 1

OR

Let the speed of man in still water be x km/h

Distance = Speed × time

$\therefore (x + 2)3 = (x - 2)6$ 1

$x = 6$

Speed of man in still water is 6 km/h 1

25. ₹ 200 is treated as interest on ₹ 1800 for 6 months.

$i = \frac{200}{1800} = \frac{1}{9}$ 1

$r_{eff} = (1 + i)^2 - 1$

$= \left(1 + \frac{1}{9}\right)^2 - 1 = 0.23456$

= 23.45% 1

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

26. $f'(x) = 6x^2 + 18x + 12$

$= 6(x + 1)(x + 2)$ 1

Increasing in $(-\infty, -2) \cup (-1, \infty)$ 1

Decreasing in (-2, -1) 1

27. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\frac{1}{2}(A + A') = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ This is symmetric 1

$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ This is skew symmetric 1

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 1

28. Evaluate: $\int \frac{(2x+1)}{(x+1)(x-2)}$ dx

$\frac{(2x+1)}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

A = $\frac{1}{3}$, B = $\frac{5}{3}$ 1

$\int \frac{(2x+1)}{(x+1)(x-2)} dx = \int \frac{1}{3(x+1)} + \frac{5}{3(x-2)} dx$ 1

$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + C$ 1

OR

Evaluate: $\int \frac{x^2}{(x-1)(x-2)(x-3)}$ dx

$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

A = $\frac{1}{2}$, B = - 4, C = $\frac{9}{2}$ 1

$\int \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} + \frac{-4}{(x-2)} + \frac{9}{2(x-3)}$ 1

$= \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + C$ 1

29. A manufacturer's total cost function C is given by C = $\frac{X^2}{25}$ + 2x

(i) Average cost function $= \frac{C}{x} = \frac{x}{25} + 2$ 1

(ii) The marginal cost function = $\frac{dC}{dx} = \frac{2x}{25} + 2$ 1

(iii) The marginal cost when 5 units are produced = 2.4 1

OR

Given that, Supply function, p = $x^2 + 4x + 5$

$p_0 = 10$

$10 = x^2 + 4x + 5$

x = -5, x = 1 1

$p_0 x_0 = 10$

Producers' surplus = $p_0 x_0 - \int_0^1 (x^2 + 4x + 5)\, dx$ 1

$= \frac{8}{3}$ 1

30. (Use $(1.025)^{20} = 1.6386$)

i = $\frac{5}{200} = 0.025$

A = $R\left(\frac{(1+i)^n - 1}{i}\right)$ 1

R = 39148 1 1

31. [Use $(1.0075)^{-300} = 0.1062$].

i = 0.0075

n = 300

$30000 = \frac{(4000000 - x) \times 0.0075}{1 - 1.0075^{-300}}$ 1 1

Down payment = ₹ 4,24,800 1

SECTION – D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. p = 0.05

n = 100

$\lambda = np = 5$ ½

$e^{-5} = 0.0067$

$P(x) = \frac{\lambda^k e^{-\lambda}}{k!}$

(i) $P(0) = e^{-5} = 0.007$ — 1 ½

(ii) $P(5) = \frac{5^5 e^{-5}}{5!} = 0.1822$ — 1 ½

(iii) P (At least one is defective) = P(0) + P(1) = 0.007 + 0.035 = 0.042 — 1 ½

OR

$\mu = 30, \sigma = 6.25$ — 1

$z = \frac{x - \mu}{\sigma}$

(i) between 20 and 40 marks

When x = 20, z = - 1.6 — 1

When x = 30, z = 1.6

P (20 < x < 40) = P (-1.6 < z < 1.6) = P (z < 1.6) – P (z < -1.6) = 0.8904 (From z table) — ½

∴ number of students scoring between 20 and 40 = 89.04% of 2000 — 1

= 1781(approximately)

(ii) less than 25 marks

When x = 25, z = - 0.8 — ½

P (z < -0.8) = 0.2119

∴ number of students scoring less than 25 = 21.19% of 2000 — 1

= 424(approximately)

33.

Year (x_i)	Index Number(Y)	$X = x_i - A$ $= x_i - 2017$	X^2	XY
2015	9	-2	4	-18
2016	18	-1	1	-18
2017	21	0	0	0
2018	29	1	1	29
2019	38	2	4	76
n = 5	$\sum y = 115$	$\sum X = 0$	$\sum X^2 = 10$	$\sum XY = 69$

2

$a = \frac{\sum y}{n} = 23$

$b = \frac{\sum XY}{\sum X^2} = 6.9$ — 1

∴ required equations is:

$y = a + bx$

$y = 23 + 6.9x$ — 1

Expected percentage for 2020:

$y = 23 + 6.9x$

$y = 23 + 6.9(2020 - 2017) = 23 + 20.7 = 43.7$ — 1

OR

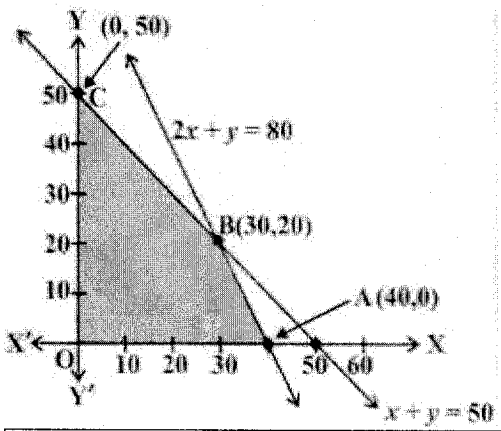
Year	Index No.	4-year moving total	4-year moving average	Centered total	Centered moving average
1980	400				
1981	470				
		1730	432.5		
1982	450			873	436.5
		1762	440.5		
1983	410			882.25	441.125
		1767	441.75		
1984	432			886.25	443.125
		1778	444.5		
1985	475			911.5	455.75
		1868	467		
1986	461			946	473
		1916	479		
1987	500			946.75	473.375
		1871	467.75		
1988	480				
1989	430				

For last 4 columns (2+1+1+1) Marks

34. Maximize: $Z = 10500x + 9000y$

Subject to constraints: $x + y \leq 50, 2x + y \leq 80, x, y \geq 0$

The corner points of the feasible region are O (0,0), A (40,0), B (30,20), C (0,50)



2

Corner Point	Value of Z
O (0,0)	0
A (40,0)	420000
B (30,20)	495000
C (0,50)	450000

2

Maximum value occurs at x = 30, y = 20

Maximum value is 495000

1

35. Solve the system of equations using matrix method

$2x - 3y + 5z = 11$, $3x + 2y - 4z = 5$, $x + y - 2z = -3$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \\ -3 \end{bmatrix}$$ 1

By Cramer's rule

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = -1$$

$$\Delta x = \begin{vmatrix} 11 & -3 & 5 \\ 5 & 2 & -4 \\ -3 & 1 & -2 \end{vmatrix} = -11$$ 1

$$\Delta y = \begin{vmatrix} 2 & 11 & 5 \\ 3 & 5 & -4 \\ 1 & -3 & -2 \end{vmatrix} = -92$$ 1

$$\Delta z = \begin{vmatrix} 2 & -3 & 11 \\ 3 & 2 & 5 \\ 1 & 1 & -3 \end{vmatrix} = -53$$ 1

$x = \frac{\Delta x}{\Delta} = 11, y = \frac{\Delta y}{\Delta} = 92, z = \frac{\Delta z}{\Delta} = 53$ 1

SECTION – E

(All questions are compulsory. In case of internal choice, attempt any one question only)

36. **Case Study – I:**

a) $k + 2k + 3k + 5k = 1$

$k = \frac{1}{11}$ 1

b) What is the probability that student studies 1 hour.

$P(1) = 2k = \frac{2}{11}$ 1

c) Calculate the mathematical expectation of number of hours studied by student.

$E(x) = 0 \times k + 1 \times 2k + 2 \times 3k + 3 \times 4k + 4 \times 5k = 40k = \frac{40}{11}$ 2

OR

What is the probability that a student study at least 3 hours

$P(x \geq 3) = 4k + 5k$ 1

$= 9k = \frac{9}{11}$ 1

37. **Case Study – II:**

a) Profit function

$P(x) = R(x) - C(x)$

$P(x) = 5x - 100 - 0.025x^2$ 1

b) $P'(x) = 5 - 0.05x$

Critical point, $x = 100$ 1

c) $P''(x) = -0.05$ 1

$\therefore$ Manufacturing 100 dolls will maximize the profit of the company,

Maximum Profit = ₹ 1,50,000 1

OR

Find the marginal cost when 250 dolls are produced. 1

Marginal cost = $C'(x) = 0.05x$

The marginal cost when 250 dolls are produced = ₹ 12.50 1

38.

a) Part of tank filled in one hour if A and C are opened together = $\frac{1}{6} - \frac{1}{12} = \frac{1}{12}$

If pipe A and C are opened together, then find the time taken

to fill the tank = 12 hours 1

b) Part of tank filled in one hour if B and C are opened together = $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

If pipe A and C are opened together, then find the time taken

to fill the tank = 24 hours 1

c) Part of tank filled in one hour if A and B are opened together = $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ 1

If pipe A and C are opened together, then find the time taken

to fill the tank = $\frac{24}{7} = 3\frac{3}{7}$ hours 1

OR

Part of tank filled in one hour if three pipes are opened together= $\frac{1}{6} + \frac{1}{8} - \frac{1}{12} = \frac{5}{24}$ 1

If three pipes are opened together, then find the time taken

to fill the tank = $\frac{24}{5} = 4\frac{4}{5}$ hours 1